Short-time dynamics of critical nonequilibrium spin models

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(Received 30 March 1998)

We study numerically the short-time critical dynamical behavior of a family of nonequilibrium models with up-down symmetry. Our numerical results show that these models, which include the Glauber dynamics as a particular case, present short-time universality. The exponent θ is calculated from the early time behavior of the time correlation of the total magnetization $\langle m(t)m(0)\rangle$ by starting from an initial state with zero correlation length and zero magnetization. [S1063-651X(98)03110-9]

PACS number(s): 05.70.Ln, 05.50.+q, 64.60.Ht

I. INTRODUCTION

Recently, a number of studies on spin models have shown that the concept of universality, which concerns the static critical behavior as well as the long-time critical relaxation, has to be enlarged to include also the critical relaxation of the early time dynamics. It has been shown by Janssen, Schaub, and Schmittmann [1], by means of renormalization group analysis, that the short-time critical relaxation displays universal behavior. Such universality has been verified on specific models such as the kinetic Ising model [2–7] and kinetic Potts model [6,8,9]. All these models have microscopic reversibility, that is, the stationary state is an equilibrium state. We may ask then whether nonequilibrium spin models may be included in the same universality class as the equilibrium models.

Nonequilibrium spin models, that is, spin models whose stationary state does not satisfy detailed balance, defined by stochastic dynamics with local rules and with up-down symmetry, were shown to be in the same universality class as the equilibrium Ising model [10]. Numerical calculations of the critical exponents on several nonequilibrium models corroborate this result [11–19]. This statement concerns only the static critical behavior and the long-time critical relaxation. The short-time critical relaxation of nonequilibrium spin models is not included in this statement but seems to be a universal behavior as found in numerical simulations on the majority vote model [20].

The purpose of the present work is twofold. The first is to present Monte Carlo simulations on a family of nonequilibrium models with up-down symmetry. Our results show that these models, which include the Glauber dynamics as a particular case, also present short-time universality. The second aim is to show that it is possible to obtain the exponent θ, related to the early time behavior of the magnetization $M(t)$, by starting from an initial state with *zero correlation length and zero magnetization*. This is the same initial state used to calculate other properties at early time such as the second moment of the magnetization and the time-dependent Binder cumulant. Usually the exponent θ is calculated by using an initial state with a small magnetization m_0 and by taking the numerical extrapolation $m_0 \to 0$.

II. TIME CORRELATION OF MAGNETIZATION

We consider here stochastic models defined on a lattice whose time evolution is governed by the master equation

$$\frac{d}{dt}P(\sigma,t)=\sum_{\sigma'} W(\sigma,\sigma')P(\sigma',t), \tag{1}$$

where W is the evolution matrix and σ denotes the collection of all Ising variables $\{\sigma_i\}$. Besides the usual properties $W(\sigma,\sigma')\geqslant 0$ for $\sigma\neq\sigma'$ and $\Sigma_\sigma W(\sigma,\sigma')=0$, the evolution matrix has the up-down symmetry $W(-\sigma,-\sigma')=W(\sigma,\sigma')$. The formal solution of Eq. (1) is

$$P(\sigma,t)=\sum_{\sigma'} T(\sigma,\sigma',t)P(\sigma',0), \tag{2}$$

where $T=\exp\{tW\}$.

The magnetization $M(t)$, defined by

$$M(t)=\frac{1}{N}\left\langle \sum_i \sigma_i(t)\right\rangle=\frac{1}{N}\sum_\sigma\left(\sum_i \sigma_i\right)P(\sigma,t), \tag{3}$$

where N is the total number of sites, should be calculated by starting with an initial state $P(\sigma,0)$ with a small magnetization m_0. If such a state is constructed by choosing the state of each site independently, then we may write

$$P(\sigma,0)=\prod_j \tfrac{1}{2}(1+m_0\sigma_j), \tag{4}$$

which up to linear terms in m_0 gives

$$P(\sigma,0)=\left(\frac{1}{2}\right)^N\left(1+m_0\sum_j \sigma_j\right). \tag{5}$$

Substituting expression (5) into Eq. (2) and this into Eq. (3) we obtain up to linear terms in m_0

$$M(t)=\sum_{\sigma\sigma'}\left(\sum_i \sigma_i\right)T(\sigma,\sigma',t)\left(\frac{1}{2}\right)^N + m_0\sum_{\sigma\sigma'}\left(\sum_i \sigma_i\right)T(\sigma,\sigma',t)\left(\sum_j \sigma'_j\right)\left(\frac{1}{2}\right)^N. \tag{6}$$

The first term on the right-hand side vanishes due to the up-down symmetry and we conclude that

$$\lim_{m_0\to 0}\frac{M(t)}{m_0}=\frac{1}{N}\sum_{\sigma\sigma'}\left(\sum_i \sigma_i\right)T(\sigma,\sigma',t)\left(\sum_j \sigma'_j\right)P_R(\sigma',0), \quad (7)$$

where $P_R(\sigma',0)=2^{-N}$ is the state with *zero correlation length and zero magnetization*. Therefore,

$$\lim_{m_0\to 0}\frac{M(t)}{m_0}=\frac{1}{N}\left\langle \sum_i \sum_j \sigma_i(t)\sigma_j(0)\right\rangle \equiv Q(t). \quad (8)$$

By using this formula in numerical simulation, we avoid the use of an initial state with a nonzero magnetization and also the numerical extrapolation $m_0\to 0$. We just start with a random generated configuration and calculate the correlation function of the total magnetization given by Eq. (8).

III. DESCRIPTION OF THE MODEL

In this paper we treat one-spin flip models so that the master equation becomes

$$\frac{d}{dt}P(\sigma,t)=\sum_i \{w_i(\sigma^i)P(\sigma^i,t)-w_i(\sigma)P(\sigma,t)\}, \quad (9)$$

where σ^i denotes the state obtained from σ by flipping the ith spin. The rate $w_i(\sigma)$ of flipping the i spin is given by

$$w_i(\sigma)=\tfrac{1}{2}\{1-\sigma_i f_i(\sigma)\}, \quad (10)$$

where $f_i(\sigma)$ is considered to be a function of the nearest neighbors of the ith spin. Moreover, by considering the model to be isotropic in space and with up-down symmetry, the most general form of $f_0(\sigma)$ for a square lattice is given by [15]

$$f_0(\sigma)=(\sigma_1+\sigma_2+\sigma_3+\sigma_4)(a+b\sigma_1\sigma_2\sigma_3\sigma_4), \quad (11)$$

where σ_1, σ_2, σ_3, and σ_4 are the nearest neighbors of σ_0. For convenience we will define the new parameters x and y by $x=2(a-b)$ and $y=4(a+b)$.

The simulation of the model is performed by choosing a spin at random. If n of the four neighboring spins have the same sign as the chosen spin, it will flip with probability p_n where $p_0=(1+y)/2$, $p_1=(1+x)/2$, $p_2=1/2$, $p_3=(1-x)/2$, and $p_4=(1-y)/2$. We consider three particular cases.

(a) The Glauber (G) model given by $y=2x/(1+x^2)$. In general the nonequilibrium models defined by Eqs. (10) and (11) do not have microscopic reversibility except when $y=2x/(1+x^2)$. In this case it is possible to write

$$f_0(\sigma)=\tanh K(\sigma_1+\sigma_2+\sigma_3+\sigma_4), \quad (12)$$

where K is related to x and y by $x=\tanh 2K$ and $y=\tanh 4K$. The rate defined by Eqs. (10) and (12) is just the Glauber dynamics whose stationary state gives the static properties of the nearest-neighbor Ising model defined on a square lattice. The critical point for this model is $K=\{\ln(\sqrt{2}+1)\}/2=0.440\,69$, which gives $x=\sqrt{2}/2=0.707\,11$ and $y=2\sqrt{2}/3=0.942\,81$.

(b) The majority vote (V) model defined by $y=x$. This model is interpreted as a collection of voters that are either in favor ($\sigma_i=+1$) or against (σ_i-1) a certain issue. At each time step a voter may change its position by looking at its four neighbors. If his position agrees with the position of the majority of the neighboring voters, its position is changed with probability $p=(1-x)/2$. Numerical results show that the critical point for this model occurs at $x=0.850\pm 0.002$ [14].

(c) The ''extreme'' (E) model defined by $x=1$. This model can also be interpreted as a collection of voters. However, in this case a given voter does not change its position if three neighboring voters have the same position as the given voter. Numerical simulations for this model shows that the critical point occurs at $y=0.710\pm 0.005$ [15].

With the exception of the G model, all other cases may also be interpreted as an Ising system in contact with two heat reservoirs at distinct temperatures [13].

IV. SHORT-TIME SCALING

According to the short-time scaling relations [1] the evolution of magnetization $M(t)$ presents a critical initial increase

$$M(t)\sim m_0 t^{\theta}, \quad (13)$$

where m_0 is the initial magnetization and θ is a new universal exponent. In order to obtain θ from this formula it is necessary to consider different small but finite values of m_0 and after make a linear extrapolation to fixed point m_0. As we have shown in expression (8) this procedure can be implicitly carried out by considering the time correlation of the total magnetization $Q(t)$ instead of the magnetization $M(t)$ and starting from a random initial configuration. So we expect the following power-law increase:

$$Q(t)\sim t^{\theta}. \quad (14)$$

We also calculate the second moment of the magnetization

$$M_2(t)=\left\langle \left(\frac{1}{N}\sum_i \sigma_i\right)^2\right\rangle, \quad (15)$$

which obeys, at the critical point, also a power law

$$M_2(t)\sim t^{\zeta} \quad \text{with} \quad \zeta=\frac{1}{z}\left(d-\frac{2\beta}{\nu}\right). \quad (16)$$

V. RESULTS

For each one of the three models, we used square lattices with sizes $L=8$, 16, 32, and 64. The total number of independent initial configurations ranged from 10^5 for $L=64$ to 4×10^5 for $L=8$. Each initial configuration was generated by setting the spin of each site up or down with equal probability and independent of each other. After that we allow the system to evolve in time according to the local rules whose parameters x and y are fixed at the critical parameters of each model. For the G model, we used the exact critical parameters $x_c=\sqrt{2}/2=0.707\,11$ and $y_c=2\sqrt{2}/3=0.942\,81$. For the V model we present results for $x_c=y_c=0.851$, and for the E model we present results for $x_c=1$ and y_c

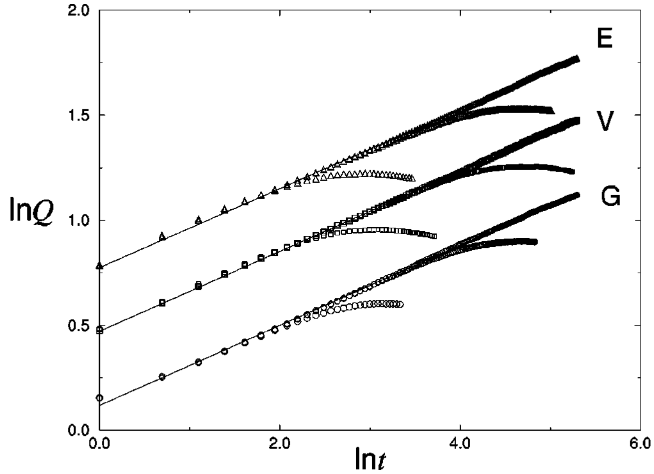

FIG. 1. Time evolution of the time correlation of magnetization $Q(t)$ for the three models G, V, and E, and for values of $L=8$, 16, and 32. The straight lines are fitted to the data points corresponding to $L=32$ and have slopes 0.191, 0.190, and 0.188, for G, V, and E, respectively. For clearness the data points corresponding to V and E were shifted in the y direction by 0.25 and 0.5, respectively.

$=0.715$. We calculated the value of the dynamical quantities, whose averages give $Q(t)$ and the second moment $M_2(t)$, at each Monte Carlo step. We repeat the same procedure for a number of initial configurations from which the averages are obtained at each time step.

In Fig. 1, we show the time evolution of $Q(t)$ defined in expression (8) for the three models G, V, and E. The slopes of the straight lines fitted to the data points give, in the plane ln$Q(t)$ versus ln$t$, the exponents $\theta=0.191$ for the G model, $\theta=0.190$ for the V model, and $\theta=0.188$ for the E model.

In Fig. 2, the second moment of the magnetization $M_2(t)$ is plotted against time, for the three above-mentioned models and for $L=64$. The slopes of the straight lines fitted to the data points give, in the plane ln $M_2(t)$ versus ln t, the exponents $\zeta=0.804$ for the G model, $\zeta=0.793$ for the V model, and $\zeta=0.790$ for the E model. Using the exact values for the

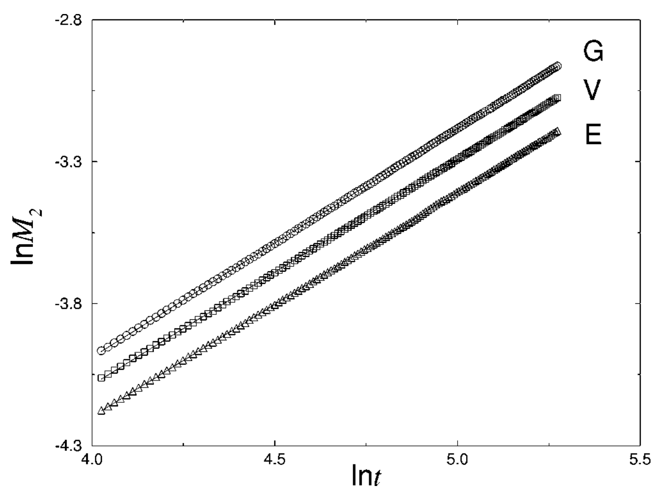

FIG. 2. The second moment of the magnetization $M_2(t)$ against time, for three models G, V, and E, for $L=64$. The straight lines fitted to the data points have slopes 0.804, 0.793, and 0.790, for G, V, and E, respectively.

exponents β and ν for the two-dimensional Ising model, namely, $\beta=1/8$ and $\nu=1$, the exponent z can be obtained from the relation $z=(d-2\beta/\nu)/\zeta$. We then get $z=2.18$ for the G model, $z=2.21$ for the V model, and $z=2.22$ for the E model.

The errors were estimated in two stages. First we measure the statistical deviations by the standard procedure in which the total measurement of a given quantity is divided in a number of statistically independent measurements. The second stage is related to the systematic errors coming from the uncertainty in the location of the critical point. The systematic errors were estimated by fixing other values of the parameters x and y that are inside the deviations around the critical values as given by previous studies of the V [14] and E [15] models. Using this procedure we obtained the values $\theta=0.190\pm0.005$ and $z=2.21\pm0.03$ for the V model and $\theta=0.188\pm0.008$ and $z=2.22\pm0.04$ for the E model. For the G model we have to consider only the statistical errors, since the critical parameters are exact, and we get the results $\theta=0.191\pm0.002$ and $z=2.18\pm0.02$.

According to Huse [2] the quantity $Q(t)$ as given by Eq. (8) behaves, at early times, as $t^{(d-\lambda_c)/z}$, which allows one to write $\theta=(d-\lambda_c)/z$ or $\lambda_c=d-\theta z$. Using our results of θ and z obtained for the G model we get $\lambda_c=1.58\pm0.01$, which agrees fairly well with his numerical calculations, $\lambda_c=1.59\pm0.02$ for $d=2$ [2].

From our results for θ and z we can also obtain the value of the exponent λ related to the short-time behavior of the autocorrelation function $\langle\sigma_i(t)\sigma_i(0)\rangle$ by using the scaling relation $\lambda=d/z-\theta=\lambda_c/z$. Using our results for the G model, we get $\lambda=0.73\pm0.01$, which agrees with the results obtained by Okano *et al.* [6].

VI. CONCLUSION

We have shown numerically that models that do not have microscopic reversibility but possess up-down symmetry have the same short-time universal behavior as the Glauber (kinetic Ising) model. The models analyzed here give values for the exponent θ that are the same within the errors. All three values are consistent with a unique universal value and our result for the G model is in excellent agreement with the results by Grassberger [21] for the Ising model. The same can be said about the exponent z obtained from the second moment of the magnetization. Our result for the Glauber model is in good agreement with the results by Grassberger [21] and Nightingale and Blöte [22] and in excellent agreement with the high-temperature series expansion, namely, $z=2.183\pm0.005$ [23]. We have also shown that the time correlation of the total magnetization $Q(t)$ defined by Eq. (8) equals the ratio $M(t)/m_0$ in the limit $m_0\to0$, which allows us to write $Q(t)\sim t^{\theta}$ for short times.

We remark that the issue of short-time universality for distinct updating algorithms has been investigated by other authors. In the case of the kinetic Ising and Potts models, studied in different lattices by different updating algorithms such as the heat-bath and Metropolis algorithms [6,7], the same short-time universal behavior was found, after a microscopic time that is different for each algorithm. With the exception of the G model, the updating rules that we have considered here describe systems whose stationary states do not satisfy detailed balance. In this case the stationary state is not known *a priori* and the models are defined only through their dynamics, that is, by the local rules and the updating procedure. Changing the updating algorithm will change the model. Despite the absence of detailed balance, short-time universal behavior is observed for models with up-down symmetry, after a microscopic time that is different for distinct models.

ACKNOWLEDGMENT

T.T. would like to acknowledge J. R. Drugowich de Felício for enlightening conversations.

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